The Prodigy That Time Forgot: The Incredible and Untold Story of John von Newton

Craig Callender

Abstract By developing an absurd counterfactual history, I show that many objections launched against Bohmian mechanics could also have been made against Newtonian mechanics. This paper introduces readers to Koopman–von Neumann dynamics, an operator-based Hilbert space representation of classical statistical mechanics. Lessons for quantum foundations are drawn by replaying the battles between advocates of standard quantum theory and Bohmian mechanics in a fictional classical history.

Born in the year 1603 in a small hamlet in the Kingdom of Hungary, John von Newton was an extraordinary polymath. It was said that when he was only six years old that he could divide two nine digit numbers in his head while conversing fluently in Ancient Greek. Widely acclaimed as the last mathematician who was equally at home in pure algebra and applied alchemy, his contributions in the Wallachian Project of the Thirty Years War led to the development of the cannon known as the Orban II. While some may know him for his development of mechanical automata, "it's-not-a-game" theory, and numerical astrology, his unparalleled advances in physics were what made him famous amongst contemporaries. However, these advances were controversial and quickly forgotten. This essay is a recounting of the astonishing breakthroughs made by John von Newton and their equally extraordinary reception.

Due to the plague in 1620, von Newton (Fig. 1) was sent home and had to study remotely. Because lessons were wrapped in straw, it was called learning by Broom. While many students suffered greatly from Broom courses and the resulting social and intellectual isolation, the circumstances had the opposite effect on a prodigy like von Newton. Finally separated from teachers and students of inferior talent, he embarked on what can only be described as the most remarkable set of intellectual leaps to ever occur in world history. In short, in six months von Newton discovered an empirically adequate (then) new physics, a theory equivalent to classical statistical mechanics, and all of the mathematical innovations necessary to express this theory

C. Callender (✉)
University of California San Diego, San Diego, CA, USA
e-mail: ccallender@ucsd.edu

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Fig. 1 John von Newton

(e.g., calculus, analysis). A month later he represented this theory with an operator formalism in a state space we now call Hilbert space.

This achievement was completed in 1620, yet by the time he died in 1699 this massive feat was forgotten. (It is speculated that the tumor that killed him may have been due to his work with toxic alchemical materials while developing Orban II.) It took until the late 19th and early 20th centuries for science to rediscover what von Newton already learned. In what follows I will summarize what he accomplished and his fate.

1 The Classical Schrödinger Equation

Contemporary writers said that von Newton would often go to bed troubled by a problem and wake up with the solution. That is why he kept a quill pen and parchment by his bedside. We don't know what problem he had in mind on the night of Feb 3, 1620, but the sepia-colored notepad from Feb 4 survives (Fig. 2). On it one can make out the faded remains of an equation that takes the following form

$$i\frac{\partial \Psi}{\partial t} = \hat{L}\Psi \tag{1}$$

when put in modern terminology. Here $\Psi(\varphi)$ is a classical wavefunction that is a ray in a complex Hilbert space. Unlike in quantum mechanics, its domain is phase space $\varphi = (q, p)$, not configuration space. The generator $\hat{L}$ is the Hermitian Louiville operator and it evolves the ray through Hilbert space with time.

Equation (1) wasn't (re)discovered until the 1930s by Koopman (1931) and a year later by von Neumann (1932). Working in the context of ergodic theory, Koopman showed that unitary transformations are central to classicla physics. In so doing

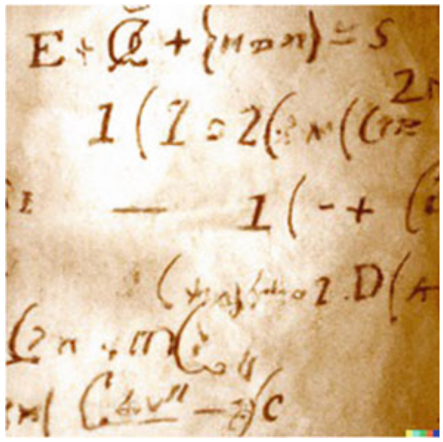

Fig. 2 Some of von Newton's calculation

he proved that if a wavefunction $\Psi(\varphi)$ satisfies (1) then the probability density $\rho(\varphi) = |\Psi(\varphi)|^2$ satisfies the classical Liouville equation $\partial_t \rho = \{H, \rho\}$, where H is the classical Hamiltonian of the system. Although this result should be widely known, apparently it is not as it has been rediscovered many times, often by very prominent physicists, e.g., Berry, Wiener, 't Hooft.[1]

von Newton's equation obviously bears a great similarity to its more famous cousin, the Schrödinger equation of quantum mechanics: $i\frac{\partial \Psi}{\partial t} = \hat{H}\Psi$. Working through von Newton's notebooks, we were astonished to see how "quantum mechanical" his formalism was. He began with four postulates:

1. The state of the system is represented by a vector $|\Psi\rangle$ in a complex Hilbert space.
2. The state space of a composite system is the tensor product of the subsystems' state spaces.
3. For any observable A, there is an associated Hermitian operator $\hat{A}$ and eigenvalue problem $\hat{A}|A\rangle = a|A\rangle$. The eigenvalue a is understood as representing a particular outcome measured in a lab.
4. The probability of measuring a is given by $P(a) = |\langle A|\,\Psi(t)\rangle|^2$. von Newton called this "Bodor's Law" .

von Newton called postulate 4 "Bodor's Law" in honor of a friend who sold the best goat milk in the hamlet. However, the name probably stuck because Bodor was renowned for his gambling prowess. von Newton interpreted Bodor's Law as arising due to an instantaneous collapse of the state $|\Psi\rangle$ into the eigenstate $|A\rangle$ associated with the measured eigenvalue a.

[1] See Berry (1992), Chirikov, Izrailev and Shepelyanskii (1988), Della Riccia and Wiene (1966), and 't Hooft (1997). The "classical Schrödinger equation" (1) should not be confused with another "classical Schrödinger equation" derived in the 1960's by Schiller (1962) and Rosen (1964). This later equation defines the wavefunction on configuration space $\Psi(q)$ whereas (1) applies to a wavefunction over phase space.

Because there is no uncertainty relation in classical physics, position and momentum have a common set of eigenstates in von Newton's Hilbert space. As mentioned, the eigenkets therefore live in phase space, not configuration space, i.e., $|A\rangle = |q, p\rangle = |q\rangle \otimes |p\rangle$. The vectors $|q, p\rangle$ form a basis of the space. By assuming what we would call a "classical commutator" $\left[\hat{q}, \hat{p}\right] = 0$ rule, von Newton was able to derive Eq. (1) from these four postulates (Bondar et al. 2021). Without modern mathematical physics at his disposal, unfortunately it took von Newton 250 pages of calculation to get this result. Helping ourselves to modern results such as the Ehrenfest Theorems and Stone's Theorem, today we can derive (1) very quickly (Wilczek 2023). Interestingly, recently (Bondar et al. 2012) show that replacing the classical commutator with the quantum commutation relations but otherwise retaining the same postulates $1 - 4$ as above leads to standard quantum mechanics. In 1620 von Newton was only one tiny adjustment from discovering quantum theory!

In any case, the resulting theory is an operator-based probabilistic theory that makes predictions about the values of measurements. The generator of motion evolves the state in a complex Hilbert space between measurements via (1) just as the Hamiltonian does in the Schrödinger equation. The norm $\langle \Psi(t)|\Psi(t)\rangle$ is conserved by the time evolution, which helps justify Bodor's Rule. And one can calculate expectation values of observables and even easily switch vector bases as one does in quantum mechanics. See Gozzi and Mauro (2004), Jordan and Sudarshan (1961), Mauro (2002), and Bondar et al. (2012) for the state of the art on Koopman–von Neumann dynamics.

As much as it looks like quantum mechanics, however, von Newton's theory was purely classical. The wavefunction lives in phase space, not configuration space. And the probabilities are the ones predicted by classical statistical mechanics, not quantum mechanics. The probabilities predicted by Bodor's Rule correspond precisely to solutions of classical statistical mechanics, i.e., the probability densities given by the classical Liouville equation. In a two slit experiment (see Mauro 2002 for a clear analysis) the phases of the classical waves cancel out and the total probability distribution on the screen is the sum of the probability distributions for each slit, reproducing what we expect classically. The theory was empirically adequate to then known empirical phenomena, which at this time consisted mostly of cannon ball trajectories.

2 Reception

When the plague ended, von Newton promoted his theory at various august academic bodies throughout Europe. With such a breathtaking set of advances, he expected to be lauded as having produced a great triumph of reason. "If I have seen further," he said, "it is because I stand as a giant." Instead the response was somewhat chilly. Scientists were impressed, but they felt uneasy about von Newton's product. His peers wanted to understand the nature of physical reality. Rene Descartes had posited a world consisting of corpuscles organized in complicated vortexes, but what was von

Newton offering? A kind of operationalist "black box" quality pervaded his theory, as his operator formalism provided only predictions for various observables.

At the University of Zurich he met a physicist named Albert Mechanstein, who would prove to be a real thorn in von Newton's side. Mechanstein was a disciple of the philosophy of Descartes. He said to von Newton that it's all well and good that you've accurately predicted the probability distribution of a bunch of cannon balls hitting a castle wall and of arrows entering sniper windows, but you don't say anything about what constitutes these balls, arrows, and walls, nor the reason *why* they behave the way they do. von Newton replied,

> Since a good theory must be based on directly observable magnitudes, I thought it more fitting to restrict myself to these.

As von Newton later recounted, Mechanstein was stunned:

> But you don't seriously believe that none but observable magnitudes must go into a physical theory?...It is the theory which decides what we can observe.

von Newton was equally upset, reporting that he was "completely taken aback by [Mechanstein]'s attitude."[2] He felt that it is "wrong to think that the task of physics is to find out how nature is"; rather, he thought, "Physics concerns what we can say about nature."[3]

As he travelled von Newton heard more objections. Pressure was put on the relationship between Bodor's Law and Eq. (1). von Newton held that we have "two fundamentally different types of interventions which can occur in a system; when an object is undisturbed, Eq. (1) "describes how the system changes continuously and causally in the course of time" but once measurement happens something "discontinuous, non-causal, and instantaneous" occurs, i.e., the collapse via Bodor's Law to an eigenstate.[4] The dynamics is deterministic when no measurement is happening, but indeterministic when it is.

This response, however, only focused attention on the role of measurement in von Newton's theory. Like standard quantum theory, von Newton's theory has a measurement problem.[5] Equation (1) is linear and allows superpositions of macroscopic outcomes; measurement collapses these superpositions to an eigenstate of

[2] Heisenberg recounting his discussions with Einstein, quoted in Becker (2018), 29.

[3] Bohr on physics after the Solvay conference, quoted in Becker (2018), 49.

[4] von Neumann describing his two dynamics, quoted in Becker (2018), 67.

[5] The two measurement problems are slightly different and interesting to consider. As Mauro (2002) emphasizes, the fundamental difference to consider. As Mauro (2002) emphasizes, the fundamental difference between Koopman–von Neumann theory and ordinary quantum theory is that in the former but not the latter the phase interacts with the modulus. Contrast a Madelung decomposition of Eq. (1) with the Schrödinger equation. Write the quantum wavefunction as $\psi(x) = A(x) exp[{}^{i}/\hbar S(x)]$and substitute it into the Schrödinger equation and then separate real and imaginary parts. Then as is well known one obtains

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + V = \frac{\hbar^2}{2mA}\frac{\partial^2 A}{\partial x^2}$$

$$m\frac{\partial A}{\partial t} + \frac{\partial A}{\partial x}\frac{\partial S}{\partial x} + \frac{A}{2}\frac{\partial^{2S}}{\partial x^2} = 0$$

the relevant observable. What interaction qualifies as a measurement, Mechanstein asked?

von Newton's acolytes differed on this question. Some said that a measurement only occurs when an outcome has been registered by a divine soul. That raised the question of who had souls. The Synod of Mâcon had long ago clarified that women had souls and Descartes compellingly argued that animals do not. But what about pagans and serfs? They certainly looked and acted like they could apply von Newton's theory as well as anyone, but did they have divine souls? And what types of souls were necessary? Was having only a vegetative soul sufficient to collapse a wavefunction?

Other acolytes did not use souls but understood measurement as an interaction between systems described in different ways. Bohr taught us that measuring devices are inherently classical, that the interaction between the classical and the quantum is central to explaining measurement. It's amazing to learn that there was a counterpart to this Bohrian position back in von Newton's day. One of his followers held that *measuring devices are essentially medieval*. What triggers a measurement is the interaction of a classical system with a medieval one, e.g., catapult, plough, water mill. Opponents felt that "medieval" was too vague to be a fundamental category in a physical theory.

A common theme emerged: scientists of the day didn't like the fundamental split between measurer and measured. Shouldn't the measurer—be they a stone mason, a nobleman, or a scythe—be itself describable in the language of physics? Why must there always be this shifty subject/object split in physics? von Newton's protestations that "for all practical purposes" it didn't matter found few sympathetic ears.

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where one can see that the phase S is coupled to the modulus A. Do the same for the classical wavefunction $\psi(x) = F(q, p)exp[i/\hbar G(q, p)]$ when inserted into (1). Then we get

$$i\frac{\partial F}{\partial t} = \hat{\mathcal{H}}\mathcal{F}$$

$$i\frac{\partial G}{\partial t} = \hat{\mathcal{H}}\mathcal{G}$$

and no coupling between F and G. (Why then introduce phases at all? They become necessary if one wants the freedom of basis one gets in Hilbert space; see Mauro 2002.) As a result of this decoupling, wavefunctions without phases cannot generate them in their time evolution. Hence the measurement problem is a bit different than quantum mechanically. In the language of foundations of physics, the classical measurement problem associated with Koopman–von Neumann is like the quantum one if decoherence worked perfectly, driving the off-diagnol terms to exactly zero. That still leaves a measurement problem, the so-called "and" to "or" problem of Bell (1990) (see also Maudlin 1995). On the classical measurement problem, see Chen (2022) (section 5.4), Katagiri (2020), and McCoy (2020).

3 Classical Mechanics Without Obs'rv'rs

What really put pressure on von Newton's theory was the remarkable development of a mechanistic theory by someone with essentially the same name, Isaac Newton. In 1687 Newton published the *Philosophiæ Naturalis Principia Mathematica*. The *Principia* posited an ontology of corpuscles who always evolve according to the same dynamical equation. Cannon balls, cannon ball parts, cannon ball operators, and cannon ball victims could all be described at once by Newton's famous second law. There was no subject/object split, no fine discussions of what types of souls or medieval devices collapse wavefunctions, or any of that. Positing one basic law rather than two, Newton offered what he called a "mechanics without obs'rv'rs."

In our age, Newton is famous for offering a physics that unified celestial and terrestrial spheres, the heavens and the earth. Back then he was also known for having provided a deeper unification of von Newton's process 1 dynamics (the deterministic Eq. 1) with von Newton's process 2 dynamics (Bodor's law). He unified the spheres *and* the two types of dynamics.

More than that, Newton offered the physical "nut-and-bolt" explanations that people didn't find in von Newton's physics. In a siege of a castle, one might shoot a cannon aimed at a wall many times. Cannon operators noticed a kind of normal statistical pattern developing on the wall. Again and again, attack after attack, similar probability distributions appeared on castle walls. Why? von Newton's physics would predict these distributions, but they couldn't answer *why* they might appear like this. It would be a very hard calculation to do, but Newton's physics at least offered one understanding of what must be going on. Small changes in the initial positions and velocities of the cannon balls, plus tiny fluctuations in their mass, are to be expected. Patterns in these differences are then responsible for why the cannon balls form these distributions.

More generally, going back to Mechanstein's complaint, the theory "decides" what is observable. That is, we can explain what is observable in terms of the posited ontology—corpuscles—and laws. We do not begin with observations as primitive, but offer explanations for why we observe what we do. These explanations were possible because Newton offered an ontology and clear laws, something that von Newton rejected.

When Newtonians ultimately derived von Newton's theory from their own, that was the death knell of the latter's influence. Suppose we have a swarm of Newtonian corpuscles sweeping out continuous trajectories through time. We can think of this as a kind of fluid described by a density $\rho(x, p, t)$. If we insist that its value is non-negative and real, it can be interpreted as the probability of a particle being at point x at time t with momentum p (using measure $\int dxdp$). It follows from Newtonian mechanics that the flow of this fluid is incompressible, which implies that

$$\frac{\partial p}{\partial t} = -\dot{x}\frac{\partial \rho}{\partial x} - \dot{p}\frac{\partial \rho}{\partial p} \tag{2}$$

holds, which provides a dynamics for ρ. Equation (1) can be derived from (2) by defining a wavefunction $\rho \equiv |\Psi(x, p)|^2$ and multiplying both sides by i (Wilczek 2023). So with Newton one could get all of von Newton's predictions but also explain *why* we were observing what we do. We could open the black box and see what's going on.

4 Criticism of Newton

von Newton and his advocates did not take these provocations lightly. They viciously attacked Newton and his physics. One giant defender of von Newton did not deign to comment on Newton's physics directly, but through intermediaries said it was "very foolish."[6] Another very distinguished physicist called it "artificial metaphysics."[7] Some took an extremely bold position (bold because manifestly false) and held that that there was no alternative to von Newton and his interpretation, that von Newton's physics "eminently possesses this character of uniqueness" in it.[8] Mostly inspired by an extreme empircist or even positivistic philosophy, these objections fell on deaf ears among the Cartesians and Newtonians of the day.

von Newton even made a political case against Newton. Like Leibniz, he wrote to Princess Caroline of Ansbach complaining about Newton's theory. Leibniz accused Newton of positing occult qualities through his non-local gravitational force and of requiring God to act as a clockmaker, fixing his product from time to time. von Newton picked up on this and also complained that Newton's clockwork universe deprived us of free will whereas his indeterministic theory made room for it. Newton was summoned before Parliament's House for Unpious Activities Committee as a result, but he answered the charges so well that no stain was left on his reputation and he was ultimately made Master of the Royal Mint.[9]

Finding fewer and fewer supporters, von Newton could only find an employment with a few of his followers at the University of Copenhagen. There he toiled in obscurity until the minstrels only sang of Newton and never the great von Newton. In some sense he had the last laugh, however, as his papers left in the gorgeous library at the University of Copenhagen were found by a young physicist named Niels Bohr.

[6] Bohr on Bohm, cited in Becker (2018), 107.

[7] Pauli on Bohm, cited in Becker (2018), 107.

[8] Rosenfeld (1957), 4–42.

[9] See Cushing (1994) for many objections to Bohm along these lines, especially by Pauli. Cushing also details the political attacks on Bohm.

5 Lessons from the Rise and Fall of von Newton

It's an honor for me to write in a volume dedicated to Detlef Dürr. He filled a room with both his warmth and knowledge. It would be impossible for me to quantify how much I learned from him and his group. One article that made a special impression is "Naive Realism about Operators", which inspired this paper. "Naive Realism..." shows in detail how the entire Hilbert space operator formalism mechanics can be derived from natural assumptions and moves from Bohmian mechanics. It argues that one should not confuse mathematical operators with physical properties of systems. Doing so leads to a fetishization of the quantum operator algebra that becomes an even bigger problem than the measurement problem.

In my absurd counterfactual history, I mimic this situation classically. I imagine that a measurement operator formalism arose first and then Newton came along with a dynamics for classical "beables" (an always determinate ontology). From this dynamics and ontology, one can then derive in detail how the entire Koopman–von Neumann Hilbert space operator formalism might arise. In the actual world, we had Newton first and Koopman–von Neumann second; and later, standard quantum theory first, Bohmian mechanics second (by only two years in the form of de Broglie). Should the temporal order of these appearances matter? I don't think so. Yet it seems almost unconscionable to launch the counterparts of the objections directed at Bohm in the actual world to Newton. Newtonian mechanics is rightly celebrated as one of the great achievements of science. While there are of course differences between the cases of Bohm and Newton, many common objections do not rely on these differences.

Since we can deduce the operator formalism of Koopman–von Neumann from Newtonian dynamics and had the latter first, we were never tempted to be "naive realists" about classical operators. But had things worked out differently, we might have been. We're often better at seeing mistakes in the past than the present, so I invented a counterfactual past and transported mistakes across times and worlds.[10]

Another lesson of the Koopman–von Neumann theory is that it is important to tease apart features of a particular mathematical representation of a theory from the theory itself. Features of a representation have a pernicious way of sneaking into our interpretation of the theory and how we evaluate it and alternatives. Jennings and Leifer (2016) ask "what phenomena of quantum theory are intrinsically non-classical?" To answer this question they apply a criterion:

> If a phenomenon of quantum physics also occurs within a classical statistical physics setting, perhaps with minor additional assumptions that don't violently clash with our everyday conceptions, then it should not be viewed as an intrinsically quantum mechanical phenomenon.

They conclude that many "commonly touted phenomena" such as randomness, complementarity, collapse of the wavepacket, the use of wavefunctions and Hilbert space, and more, cannot be marks of intrinsically quantum phenomena. I wholeheartedly agree. By placing classical statistical mechanics in an operator-based formalism in

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[10] See Nikolić (2008) for a less incredible counterfactual history toward the same point.

Hilbert space, Koopman and von Neumann demonstrably show that many of these representational features are not inherently quantum mechanical. Not only is classical physics expressible in a similar formalism, but it can also employ collapses of the wavefunction, Born's Rule, a fundamental subject/object split, and two types of dynamics. The operator measurement formalism seems to almost invite an interpretation with an instrumentalist flavor.

If one is a naive realist about classical observables, Koopman–von Neumann even has a measurement problem. But that is the result of a choice, a bad choice. Classical statistical mechanics does not have a measurement problem. Neither does quantum mechanics if one adopts a decent interpretation, e.g., the Bohmian mechanics that Detlef prized. The measurement problem in Koopman–von Neumann makes this point plain. There it results not from the peculiarities of the classical world but from the peculiarities of "quantum philosophy" applied to the classical world. As Detlef saw much better than most, the same is true in quantum physics. That is the ultimate lesson of the tragedy of the great and forgotten John von Newton, the naive realist about classical observables.

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